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The limits for u are 0 and $\pi/2$, and as the equation of the given plane is $\tan t = \tan \alpha \sin u$, the limits for t will be $\tan^{-1}(\tan \alpha \sin u)$ and $\pi/2$.

Therefore

$$S = \frac{\sqrt{a^2 + b^2}}{2} \int_0^{\pi/2} du \int_{\tan^{-1}(\tan \alpha \sin u)}^{\pi/2} (a + a \cos 2t + b \sin 2t) dt.$$

2772 [1919, 171]. Proposed by HARRY LANGMAN, New York City.

Given $1 = (-\frac{1}{2} + x)^r = (-\frac{1}{2} - x)^r$, where r is integral. Prove that r is a multiple of 3. In general, if

$$1 = \left[\cos \frac{2\pi}{m} + x \right]^r = \left[\cos \frac{2\pi}{m} - x \right]^r,$$

where r and m are integers, prove that r is a multiple of m .¹

SOLUTION BY THE PROPOSER.

The general case only will be considered, and the trivial case $r = 0$ excluded.

Put

$$(1) \quad a = \cos \frac{2\pi}{m} + x \quad \text{and} \quad b = \cos \frac{2\pi}{m} - x.$$

Then

$$(2) \quad a + b = 2 \cos \frac{2\pi}{m}.$$

We may write

$$(3) \quad 1 = \cos 2k\pi + i \sin 2k\pi.$$

Hence,

$$(4) \quad a = \cos \frac{2k\pi}{r} + i \sin \frac{2k\pi}{r} \quad \text{and} \quad b = \cos \frac{2k'\pi}{r} + i \sin \frac{2k'\pi}{r}, \quad 0 < k, k' < r + 1.$$

From (2),

$$\cos \frac{2k\pi}{r} + \cos \frac{2k'\pi}{r} + i \left(\sin \frac{2k\pi}{r} + \sin \frac{2k'\pi}{r} \right) = 2 \cos \frac{2\pi}{m},$$

from which

$$(5) \quad \cos (k + k') \frac{\pi}{r} \cdot \cos (k - k') \frac{\pi}{r} = \cos \frac{2\pi}{m} \quad \text{and} \quad \sin (k + k') \frac{\pi}{r} \cdot \cos (k - k') \frac{\pi}{r} = 0.$$

Hence, we must have

$$(6) \quad \sin (k + k') \frac{\pi}{r} = 0.$$

From the problem, if $m \neq 1$, we must have $a \neq b$. Hence, in (4) $k \neq k'$. Hence, we must have $k + k' < 2r$, that is, $(k + k')/r < 2$. But, from (6), $(k + k')/r$ must be integral. Hence, since $k + k' > 0$,

$$(7) \quad k + k' = r \quad \text{and} \quad k - k' = 2k - r.$$

From the first equation of (5), we then obtain (8) $\cos \frac{2\pi}{m} = -\cos \frac{2k - r}{r} \pi$.

Now $\frac{2}{m} \leq 1$ and $\left| \frac{2k - r}{r} \right| \leq 1$. Therefore, from (8),

$$\left| \frac{2k - r}{r} \right| = 1 - \frac{2}{m}.$$

Hence, $\frac{2k}{r} - 1 = 1 - \frac{2}{m}$ or $1 - \frac{2k}{r} = 1 - \frac{2}{m}$, from which $k = r - \frac{r}{m}$ or $k = \frac{r}{m}$. From (7),

$k' = \frac{r}{m}$ or $k' = r - \frac{r}{m}$. Since k and k' are integral, we must have $\frac{r}{m}$ integral. Hence, r is a multiple of m .

¹ If $m = 4$ we may take $x = 1, r = 2$; in this case r is not a multiple of m . Therefore the theorem of the question is not true for this case.—EDITORS.

Also solved by C. A. BARNHART, H. HALPERIN, H. L. OLSON, and A. PELLETIER.

2773 [1919, 212]. Proposed by JOSEPH ROSENBAUM, Milford, Conn.

Point out the fallacy in the proof of the following problem:

In the triangle $A_1B_1C_1$, let m be a point such that the sum of the distances from it to the sides is a maximum; also let $A_2B_2C_2$ be a triangle formed by drawing lines through the vertices A_1, B_1 , and C_1 parallel to their opposite sides. Then the sum of the distances from m to the sides of the triangle $A_2B_2C_2$ is a minimum.

Proof.—Because the sides of the two triangles are parallel in pairs, the sum of the distances from a variable point P in triangle $A_1B_1C_1$ to the six sides of the two triangles is constant. Now by hypothesis M is a point for which one part of this constant sum is a maximum, and hence it follows that the other part is a minimum.

SOLUTION BY H. L. OLSON, University of Wisconsin.

This proof is correct, with the understanding that if a point P is on the opposite side of BC , for example, to the vertex A , the distance to the side BC is to be regarded as negative. It is easy to see, however, that the point M does not exist, and that the proposition is therefore vacuous. Represent the perpendicular distances from P to the sides BC, AC , and AB by α, β , and γ respectively. If we denote by Δ the area of the triangle ABC , we are to minimize the function $\alpha + \beta + \gamma$, subject to the condition $a\alpha + b\beta + c\gamma = 2\Delta$. (a, b , and c represent, as is customary, the sides BC, AC , and AB , respectively.) Eliminating γ , we have, as the function to be minimized,

$$\left(1 - \frac{a}{c}\right)\alpha + \left(1 - \frac{b}{c}\right)\beta + \frac{2\Delta}{c}.$$

Hence, the derivatives, $\left(1 - \frac{a}{c}\right)$, and $\left(1 - \frac{b}{c}\right)$, of this function with respect to α and β

must vanish; but for the general triangle they do not vanish and hence M does not exist. If, however, $a = b = c$, the sum of the distances is the constant $2\Delta/c$; likewise the sum of the distances for the corresponding triangle $A_2B_2C_2$ is constant.

Also solved by A. PELLETIER and A. L. WECHSLER.

2774 [1919, 212]. Proposed by FRANK IRWIN, University of California.

Evaluate the circulants

$$\begin{vmatrix} 1 & 2 & 3 & \cdots & n-1 & n \\ n & 1 & 2 & \cdots & n-2 & n-1 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ 2 & 3 & 4 & \cdots & n & 1 \end{vmatrix}, \quad \text{and} \quad \begin{vmatrix} a_1 & a_2 & a_3 & \cdots & a_{n-1} & a_n \\ a_n & a_1 & a_2 & \cdots & a_{n-2} & a_{n-1} \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot \\ a_2 & a_3 & a_4 & \cdots & a_n & a_1 \end{vmatrix},$$

where, in the latter, a_1, a_2, \dots, a_n form an arithmetical progression.

I. SOLUTION BY P. J. DA CUNHA, University of Lisbon, Portugal.

Denote the first of these circulants by Δ and the second by Δ^4 . Let

$$s_n = \frac{1+n}{2}n$$

be the sum of the first n positive integers. Add to the elements of the last line of Δ the sum of the corresponding elements of all the preceding lines. We obtain a determinant which we can write as the product

$$\Delta = s_n \begin{vmatrix} 1 & 2 & 3 & \cdots & n-2 & n-1 & n \\ n & 1 & 2 & \cdots & n-3 & n-2 & n-1 \\ n-1 & n & 1 & \cdots & n-4 & n-3 & n-2 \\ \cdot & \cdot & \cdot & \cdots & \cdot & \cdot & \cdot \\ 4 & 5 & 6 & \cdots & 1 & 2 & 3 \\ 3 & 4 & 5 & \cdots & n & 1 & 2 \\ 1 & 1 & 1 & \cdots & 1 & 1 & 1 \end{vmatrix}.$$